Central Limit Theorem for Means

Lecture 30
Section 8.4

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Outline

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The Central Limit Theorem for Means

Theorem (Central Limit Theorem for Means (simple version))

Let a population have mean μ and standard deviation σ. The population of sample means $\overline{x}$ from all possible samples of size n has a normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

- We will do an experiment to put this to the test.

Outline

The Population

- Suppose a population consists of 100 people, 20% of whom weigh 60 lbs, 20% of whom weigh 120 lbs, and 60% of whom weigh 180 lbs.
- Then the mean of the population is

$$\mu = 144$$

and the standard deviation is

$$\sigma = 48.$$

The Population

- If we take many samples of size $n = 10$ and compute their sample means, we should see a normal distribution with mean 144 and standard deviation $\frac{48}{\sqrt{10}} = 15.1789$.

The Population

Person	Wgt	Person	Wgt	Person	Wgt	Person	Wgt	Person	Wgt
1	180	21	180	41	180	61	180	81	60
2	180	22	120	42	180	62	180	82	180
3	60	23	180	43	120	63	60	83	60
4	180	24	120	44	60	64	180	84	180
5	120	25	180	45	60	65	180	85	120
6	180	26	60	46	120	66	180	86	60
7	180	27	120	47	180	67	180	87	180
8	180	28	180	48	180	68	180	88	180
9	120	29	60	49	60	69	120	89	120
10	60	30	120	50	180	70	60	90	180
11	120	31	180	51	60	71	180	91	180
12	180	32	180	52	180	72	180	92	180
13	60	33	180	53	120	73	180	93	180
14	180	34	180	54	180	74	60	94	180
15	180	35	180	55	180	75	180	95	120
16	120	36	180	56	180	76	180	96	60
17	180	37	180	57	120	77	120	97	180
18	180	38	180	58	60	78	180	98	180
19	60	39	60	59	180	79	180	99	120
20	60	40	180	60	120	80	120	100	180

An Experiment

- Randomize your TI-83. Use the last 4 digits of your phone number as a seed, e.g., `8604→rand`.
- Use the TI-83 to get 10 samples of 10 people each. (Allow repetitions.)
- For each sample, count how many times each weight (60, 120, 180) occurred.
- Compute the total. For example, if 60 occurred 3 times and 120 occurred 1 time and 180 occurred 6 times, then compute

$$\begin{aligned} 3(60) + 1(120) + 6(180) &= 180 + 120 + 1080 \\ &= 1380. \end{aligned}$$

- Divide by 10 to get the sample mean of 138.
- When you are finished, report the 10 sample means that you found.

An Example

- For example (with seed 8604),

No.	Sample	# 60s		# 120s		# 180s		Mean
1	$\{81, 17, 100, 25, 29, 69, 87, 90, 57, 6\}$	$\|\|$	120	$\|\|$	240	$\text{𝍸}\,\|$	1080	144
2	$\{51, 85, 18, 32, 94, 17, 76, 63, 81, 98\}$	$\|\|\|$	180	$\|$	120	$\text{𝍸}\,\|$	1080	138
3	$\{21, 71, 30, 5, 91, 29, 11, 47, 37, 99\}$	$\|$	60	$\|\|\|\|$	480	𝍸	900	144
4	$\{21, 83, 47, 21, 84, 81, 54, 89, 92, 10\}$	$\|\|\|$	180	$\|$	120	$\text{𝍸}\,\|$	1080	138
5	$\{22, 79, 51, 98, 8, 11, 20, 13, 44, 36\}$	$\|\|\|\|$	240	$\|\|$	240	$\|\|\|\|$	720	120
6	$\{86, 17, 2, 28, 15, 38, 93, 95, 21, 42\}$	$\|$	60	$\|$	120	$\text{𝍸}\,\|\|\|$	1440	162
7	$\{82, 47, 43, 9, 100, 53, 91, 54, 23, 97\}$		0	$\|\|\|$	360	$\text{𝍸}\,\|\|$	1260	162
8	$\{62, 26, 17, 57, 26, 1, 58, 76, 81, 91\}$	$\|\|\|\|$	240	$\|$	120	𝍸	900	126
9	$\{79, 95, 52, 61, 25, 37, 21, 26, 52, 52\}$	$\|$	60	$\|$	120	$\text{𝍸}\,\|\|\|$	1440	162
10	$\{61, 62, 78, 75, 35, 45, 50, 5, 62, 66\}$	$\|$	60	$\|$	120	$\text{𝍸}\,\|\|\|$	1440	162